Abstract

An energy recovery linac (ERL) is a possible candidate for an upgrade of the Advanced Photon Source (APS). Our ERL design includes full-energy linac, large turn-around arc that could accommodate new x-ray beamlines, and APS itself. In total, the beam trajectory length would be close to 3 km. The ERL lattice has a strong focusing to limit emittance growth, and it includes strong sextupoles to keep beam energy spread under control and minimize beam losses. As in storage rings, trajectory errors in sextupoles will result in lattice perturbations that would affect delivered x-ray beam properties. In storage rings, the response matrix fit method is widely used to measure and correct linear lattice errors. Here, we explore the application of the method to the linear lattice correction of ERL.

INTRODUCTION

Linear optics measurement and correction using response matrix fit is well known and widely used on modern circular machines. The purpose of this work is to simulate the application of the same method to a non-closed beamline.

Theoretically, there is no big difference between response matrix measurement for closed and non-closed beamlines. The orbit equations are well-known and look similar (top equation is for non-closed trajectory and bottom is for closed trajectory, \( \theta \) – is the kick strength):

\[
x(s) = \theta \sqrt{\beta_x \beta_y} \sin(\psi_x - \psi_y),
\]

\[
x(s) = \frac{\theta}{2 \sin(\pi \nu)} \sqrt{\beta_x \beta_y} \cos(\psi_x - \psi_y - \pi \nu).
\]

The measured trajectories in both cases depend on beta functions and phase advances and therefore could be used to derive linear optics. The main practical difference is that in the case of non-closed beamline, the response matrix is triangular with zeros in the top right triangle.

SIMULATION DETAILS

At APS, we have been using response matrix fit method for many years [1]. We added an option of working with non-closed trajectories to our existing program. From our experience, we know that at APS the main source of focusing errors are non-zero orbits in sextupoles and we also know that the focusing errors from sextupoles cannot be precisely represented by nearest quadrupoles [2]. Therefore we decided to include sextupole displacements in error simulation. The following set of errors was used for simulations:

\begin{align*}
\text{Quadrupole gradient error} & \quad 0.1 \% \\
\text{Quadrupole tilt} & \quad 0.001 \text{ rad} \\
\text{Sextupole X and Y displacement} & \quad 1 \text{ mm} \\
\text{Corrector calibration error} & \quad 5 \% \\
\text{Corrector tilt} & \quad 0.001 \text{ rad} \\
\text{BPM calibration error} & \quad 2 \% \\
\text{BPM tilt} & \quad 0.001 \text{ rad} \\
\text{BPM measurement noise} & \quad 1 \text{ \( \mu \)m}
\end{align*}

Sextupole displacements were chosen rather large because trajectory errors in sextupoles are defined not by the accuracy of sextupole alignment but by the accuracy of nearest BPM offset which could be large. The errors were generated using Gaussian distribution with 2 sigma limit.

For optics correction simulation we used only APS portion of the ERL because the Turn-Around Arc design has not been finalized to a level of BPM and corrector locations. The lattice of the APS is presented in Figure 1. The main difference from the present APS storage ring lattice is zero dispersion in ID straight sections to decrease electron beam size dependence on energy spread. The APS consists of 40 nearly identical sectors.

Figure 1: Lattice functions of one sector of the APS portion of the ERL.

Special attention was paid to the choice of correctors used for response matrix measurement in our simulations. APS storage ring has 8 correctors and 11 BPMs per sectors (in most sectors). Presently, for real measurements we use only 27 correctors in each plane (out of 320) evenly distributed along the ring and all BPMs. We limit the number of correctors in order to save measurement time and also to limit the size of the fitting problem. If all the correctors were used, the size of the response matrix derivative would be 15 Gb, which would be too big. Our experience shows that with 27 correctors we still have...
enough data for an accurate fit. In case of a circular machine, the location of correctors used for the response matrix measurement is not important as long as they are separated by some phase advance. However, the situation is different for a non-closed beamline where measured trajectory is affected only by elements that are located after the steering magnet. Therefore, for a non-closed beamline, different steering magnets provide different amount of useful information. Obviously, one would want to use as many steering magnets in the beginning of the beamline as possible while keeping them at some phase space distance. For our simulations, we used 27 correctors in each plane spread over first six sectors (out of forty) and none after that.

The following procedure was used to simulate the entire process of measurement and optics correction (elegant [4] was used for all beta function and trajectory calculations):

- elegant parameter file is generated with element errors;
- trajectory is corrected (because the sextupole displacements could lead to large trajectory errors) using 2 correctors per sector;
- “measured” response matrix and dispersion are calculated on the corrected orbit, response matrix is generated from two trajectories for each corrector using the same plus-minus delta approach that we use in real measurements;
- response matrix fit is calculated (dispersion included);
- quadrupole gradient errors opposite to those found in the response matrix fit are applied to correct the optics, and the resulting beta functions are compared with the ideal beta functions.

The entire process was run 100 times with different error seeds. Figure 2 shows typical beta functions before beta function correction. For each case, we have calculated relative beta function difference between actual and ideal beta functions and its rms value (the rms value is calculated using all beta function points along the beamline). Figure 3 shows histogram of rms of relative beta function errors before correction. Average rms of the relative beta function difference over all cases is 0.71 for horizontal and 0.47 for vertical plane.

**CORRECTION RESULTS**

All APS quadrupole magnets have separate power supply. Therefore, the straightforward way to correct the optics is to apply opposite quadrupole gradients. However, this method has some drawbacks that prevent us from using it in real life. To achieve the best possible response matrix fit, we use as many singular values in matrix inversion as possible. This might lead to appearance of large quadrupole errors in the solution. After we calculated beta functions using quadrupole errors from the response matrix fit, we use inverse beta function response matrix to correct the difference between measured and ideal beta functions. We also adjust the number of singular values in this inversion until we get satisfactory correction accuracy while still keeping quadrupole changes small. This allows us to minimize real quadrupole changes during optics correction at APS storage ring.

![Figure 2: Typical beta functions before beta function correction. Top left – horizontal, top right – vertical, and bottom is dispersion.](image)

![Figure 3: Histogram of the relative beta function error rms before beta function correction. Histogram is calculated over the set of 100 different error seeds. For every seed, the relative beta function error was calculated, and then rms was calculated using all beta function points along the beamline.](image)

However, these arguments are not important for the optics correction simulation here, so we used the straightforward approach to keep our simulations simple. Figure 4 shows histogram of rms of relative beta function errors after correction. Average rms of the relative beta function difference over all cases is 0.03 for horizontal and 0.02 for vertical plane. Figure 5 shows typical lattice functions after correction.

We can estimate the effect of residual dispersion perturbation on the beam size using typical ERL parameters: $\varepsilon=10$pm and $\sigma_E=0.02\%$. Maximum beam size increase at ID location due to energy spread contribution using dispersion on Figure 5 (bottom plot) is about 15%. If such accuracy of dispersion correction turns out to be not satisfactory, it can be corrected separately afterwards.
since the dispersion (unlike beta functions) can be directly measured by scanning beam energy.

Figure 4: Histogram of the relative beta function error rms after beta function correction. See Figure 3 for comment on plot units.

One can ask why the correction is not perfect. Two reasons are obvious – due to BPM noise the response matrix measurement is not accurate and due to the fact that the focusing errors come from sextupoles but are corrected using quadrupoles in different locations. These reasons are likely to explain short-scale perturbations in beta functions. But we can also see a long-scale smooth variation in the horizontal beta function on Figure 5. The reason for that is inaccurate determination of focusing errors in the very beginning of the lattice because the first quadrupoles and sextupoles have only few trajectories going through them. If this argument is true, then the beta function variation can be corrected by adjusting initial beta functions at the entrance of the lattice. Figure 6 shows horizontal beta functions that were obtained by varying incoming beta function and its slope. A small change in initial beta function conditions allows correcting the long smooth variation seen on top left plot of Figure 5 thus confirming that quadrupole errors in the beginning of the beamline were not determined correctly. To improve accuracy for first elements of the measured beamline, one might use several correctors upstream of measured portion of the beamline.

CONCLUSION

We have simulated optics correction for non-closed beamline using response matrix fit. As example we used suggested APS lattice in ERL mode. We have found that response matrix fit can be used to measure and correct linear lattice successfully. We have confirmed that one can measure and correct only a part of non-closed beamline which will be useful for large ERLs.

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REFERENCES