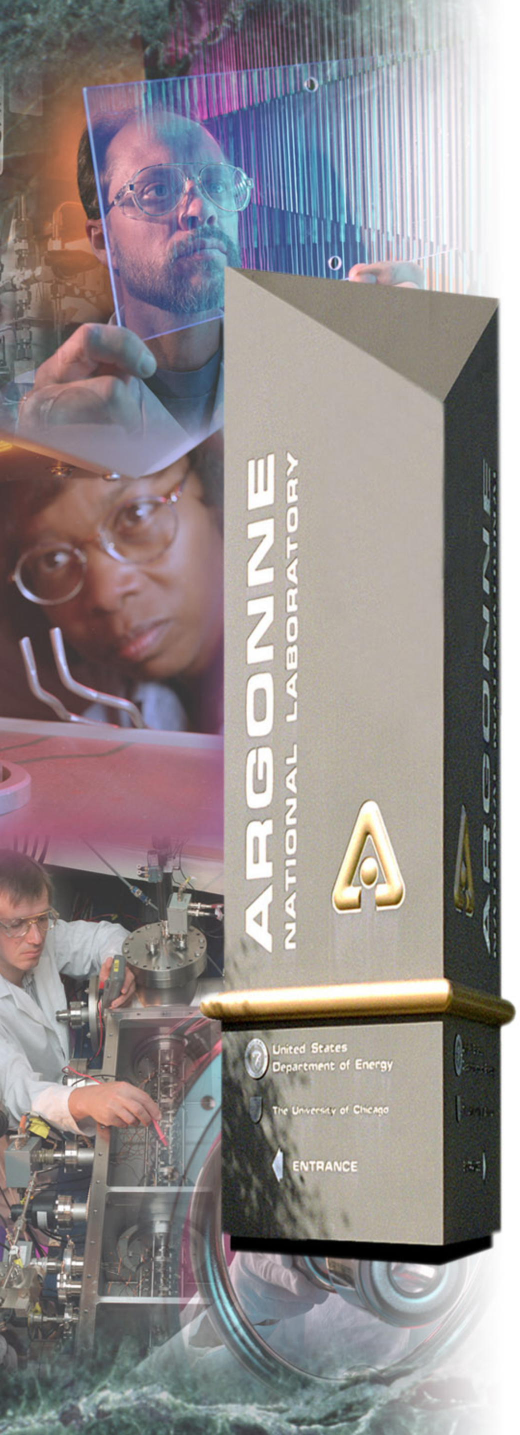

Sextupole Optimization for Deflecting Cavity Scheme

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Acknowledgments:
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How sextupoles affect the beam

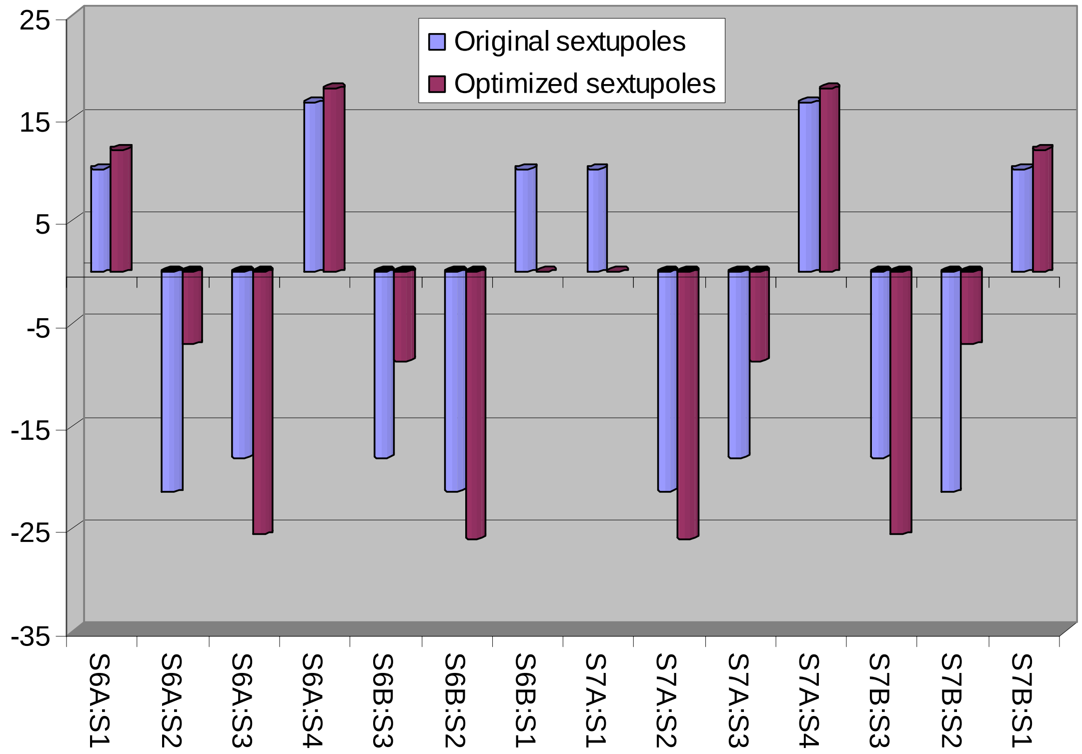
- In first order, particles traveling with non-zero vertical trajectory through a sextupole see additional skew quadrupole field
- That creates coupling between planes and therefore vertical emittance increase
- Sextupoles are located in non-zero horizontal dispersion so vertical dispersion will also be excited through skew quadrupole field
- Higher-order effects can also be important (nonlinear coupling in sextupoles)

Optimization procedure

- **Direct optimization based on one-pass tracking results through the deflecting section using `elegant`**
- **Constraints are:**
  - Minimize vertical emittance increase
  - Compensate chromaticities to zero
- **Variables are:**
  - All sextupoles between cavities symmetrically around the center of the deflecting section
- **Variable limits:**
  - Maximum sextupole gradient is increased by 25%
  - Sextupole signs are kept constant

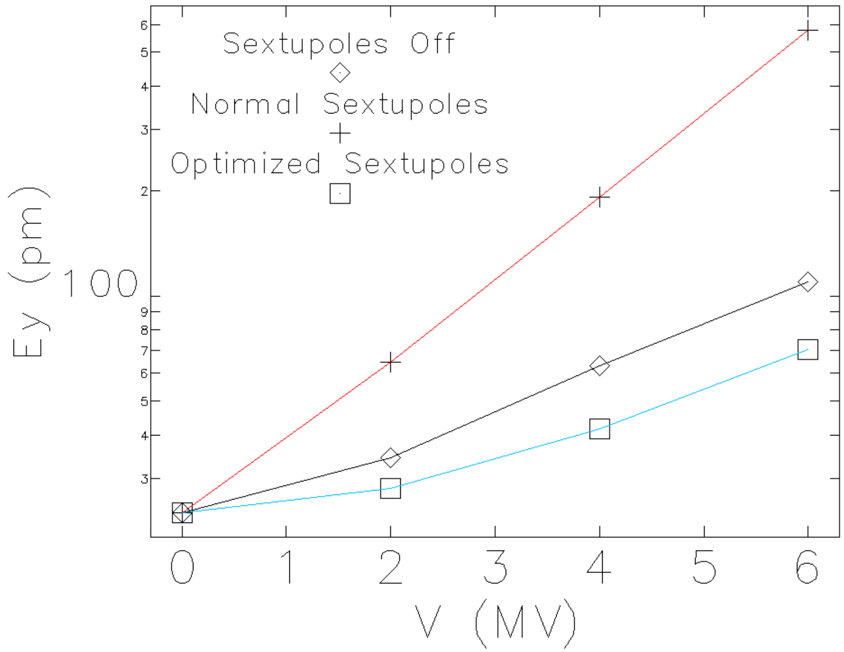
Optimization results

- **Optimized sextupole strengths (all sextupoles between cavities):**

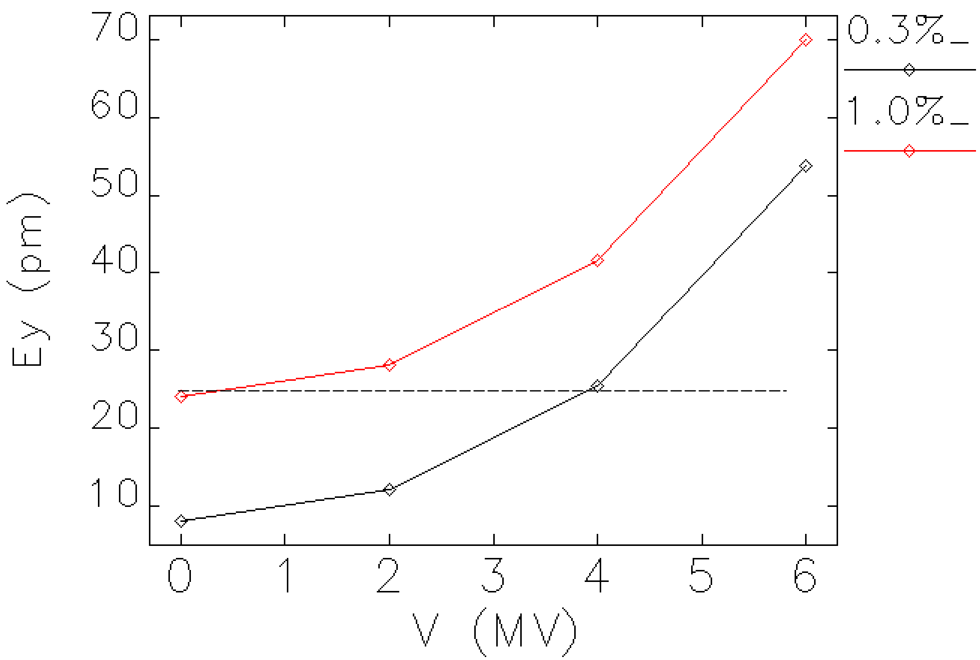


Optimization results (2)

Comparison of the three sextupole schemes (no synchrotron radiation)

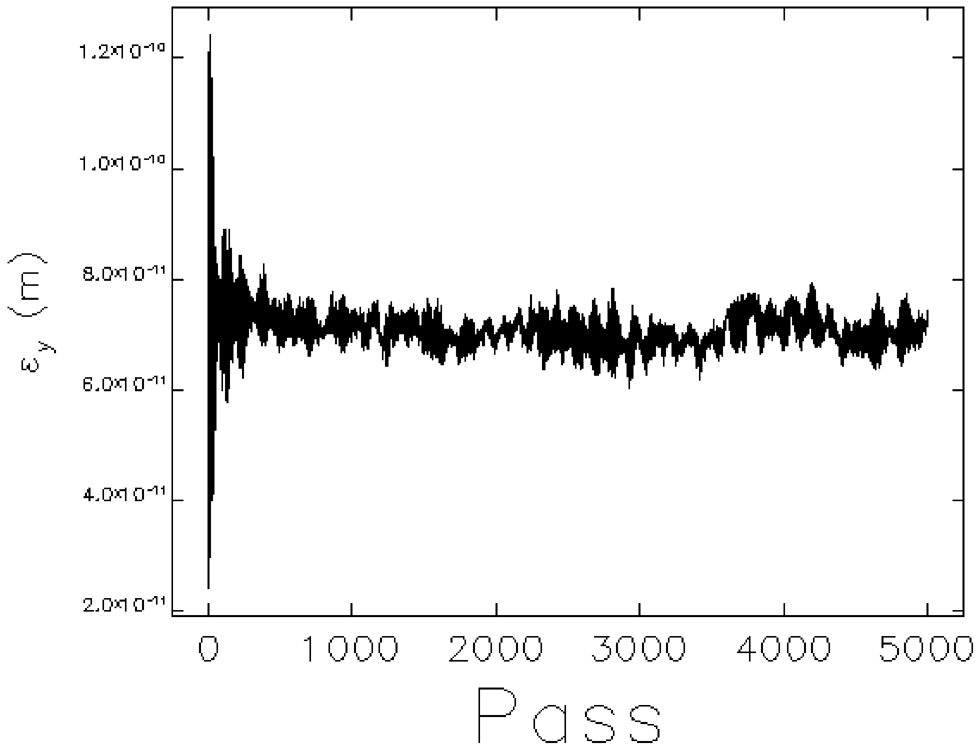


Example of lower coupling with optimized sextupoles



Optimization results (3)

Previous studies have shown that synchrotron radiation can greatly affect the tracking results. Here we show that simulation with synchrotron radiation does not change the results.



Analysis: Coupling harmonic compensation

- **The degree of emittance coupling depends on the tunes and the coupling coefficient:**

$$\kappa_q = \frac{1}{2\pi}\int_0^C \mathrm{K}_s \sqrt{\beta_x \beta_y}\, e^{i\Psi_q} ds\ ,$$

$$\Psi_q = \psi_x - \psi_y - (\nu_x - \nu_y - q)\theta$$

- **The value of coupling harmonic for the slice experiencing 100 μrad kick:**
  - **Normal sextupoles:** $\kappa_{17} = 4.6 \cdot 10^{-2}$
  - **Optimized sextupoles:** $\kappa_{17} = 1.6 \cdot 10^{-2}$

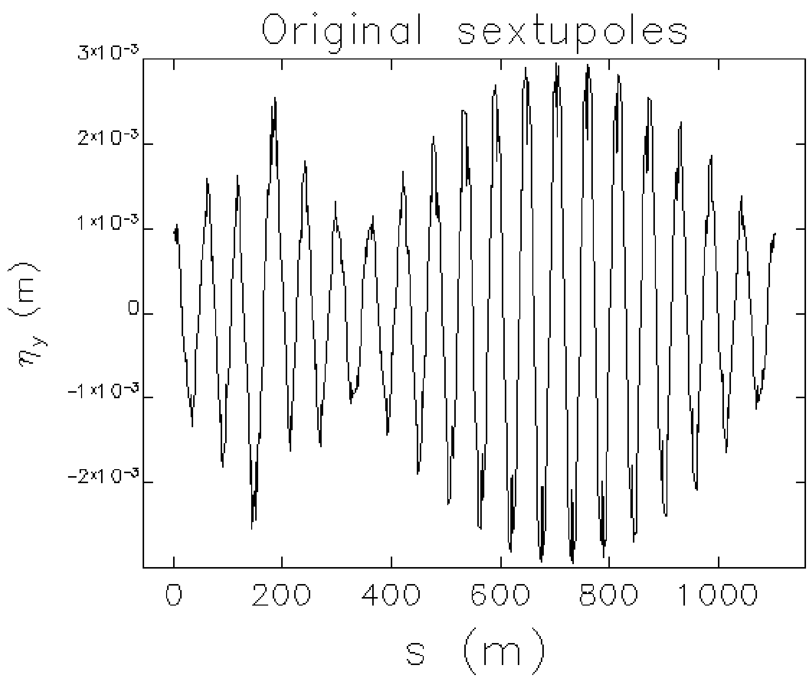
Analysis: Off-diagonal matrix elements

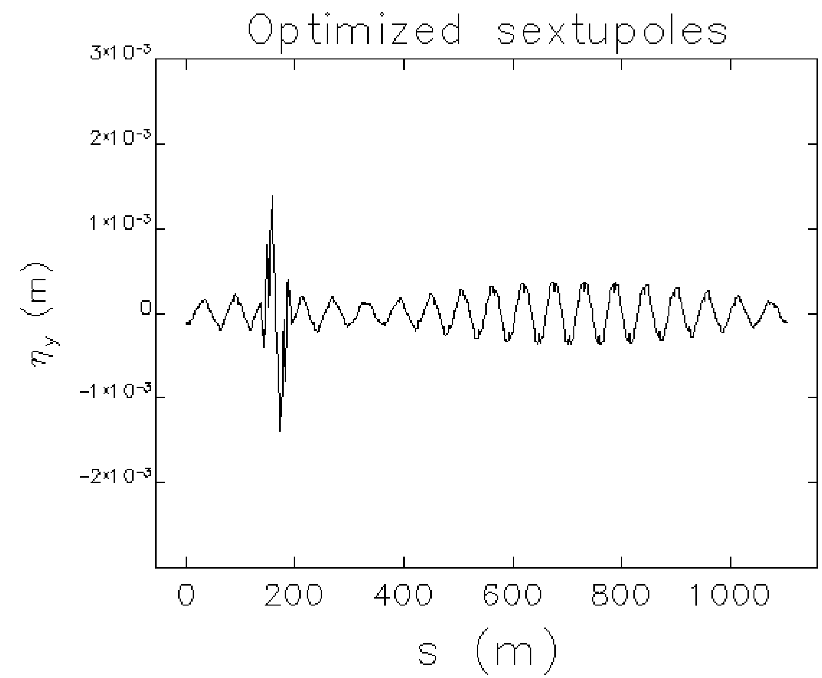
- **We define 4×4 transformation matrix between cavities as follows:**

$$M = \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix}$$

- **Coupling can be quantified by the determinant of M_{xy}:**
  - **Normal sextupoles:** $\left| M_{xy} \right| = 4.6 \cdot 10^{-4}$
  - **Optimized sextupoles:** $\left| M_{xy} \right| = 8.3 \cdot 10^{-6}$

Analysis: Vertical dispersion compensation





Analysis: Tune shift with amplitude

- **We define tune shift with amplitude as follows:**

$$\delta\nu_x = C_{xx} J_x + C_{xy} J_y + o(J^2)\,,$$
$$\delta\nu_y = C_{xy} J_x + C_{yy} J_y + o(J^2)\,,$$

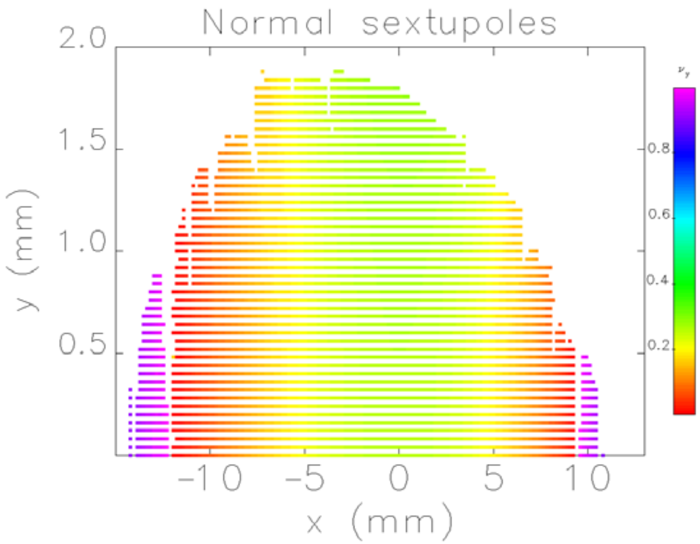
- **When calculated for the entire ring, tune shift with amplitude does not change significantly**
- **Tune shift calculated for deflection section only:**
  - **Normal sextupoles**

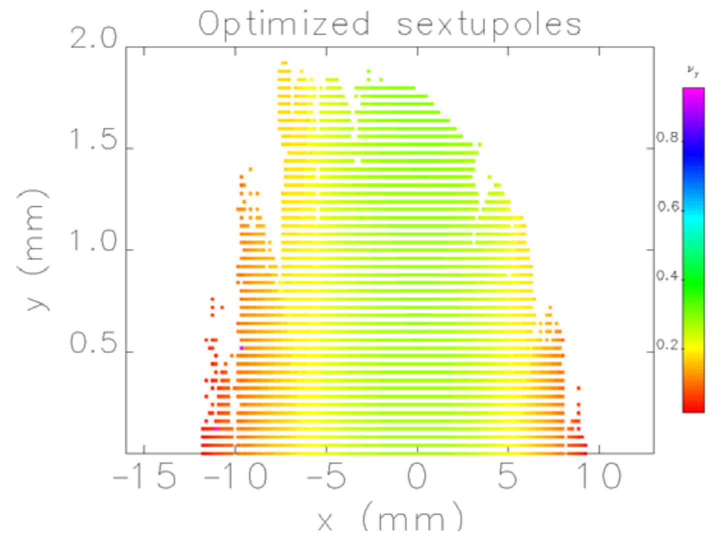
$$C_{xx} = -718\,{}^{1}\!/_{m}\,,\quad C_{xy} = 2215\,{}^{1}\!/_{m}\,,\quad C_{yy} = -1600\,{}^{1}\!/_{m}\,.$$

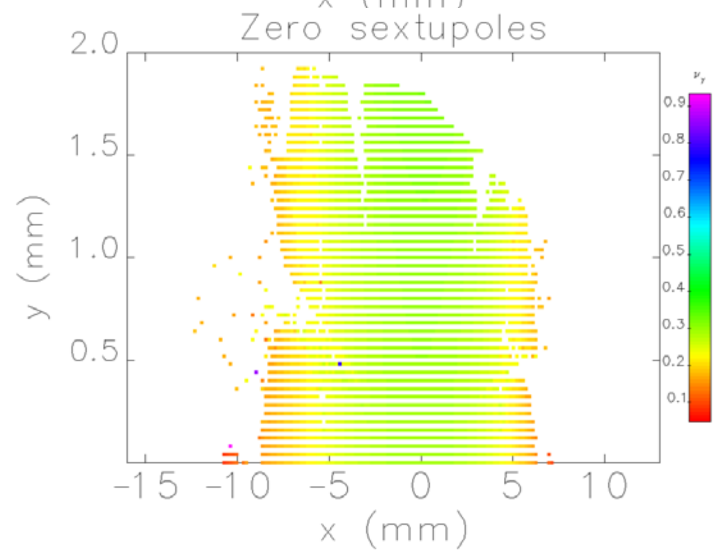
  - **Optimized sextupoles**

$$C_{xx} = 1960\,{}^{1}\!/_{m}\,,\quad C_{xy} = -900\,{}^{1}\!/_{m}\,,\quad C_{yy} = 880\,{}^{1}\!/_{m}\,.$$

Dynamic aperture comparison







Lattice without errors

500 turns tracking

Color indicates vertical tune

Expansion to more than 2 sectors

- Optimization of sextupoles opens possibility to increase the number of sectors that could benefit from the compression scheme

Number of sectors	Vertical emittance
2	70 pm
3	59 pm
4	41 pm

- Vertical emittance blowup is no longer a limitation. Instead, new limit would be dynamic aperture decrease

Conclusions

- Due to proper optimization of sextupole strength, the vertical emittance increase is no longer a limiting issue for this scheme.

- It seems possible to increase the number of sectors between cavities to more than two. That would require additional dynamic aperture study, which is underway.