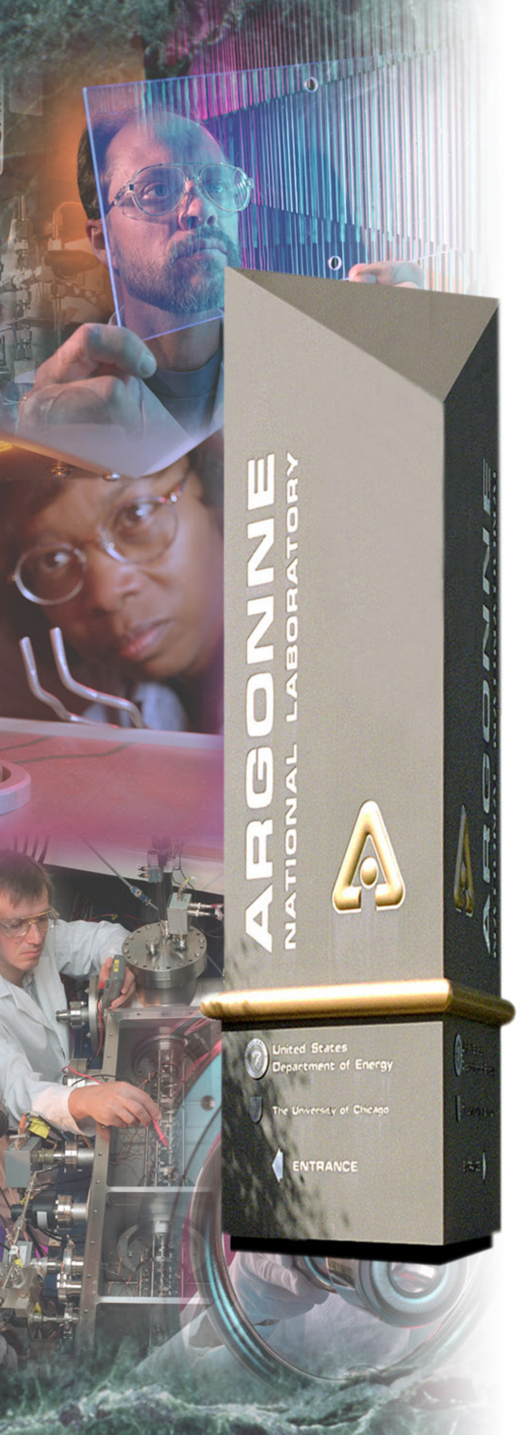

Containing a blow-up of vertical emittance in the RF deflection scheme for a generation of sub-picosecond X-ray pulses

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Acknowledgments:
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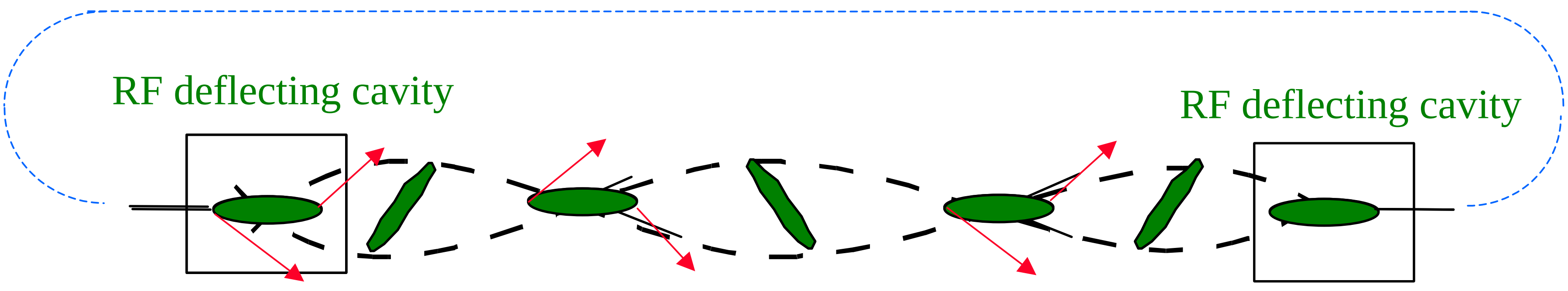
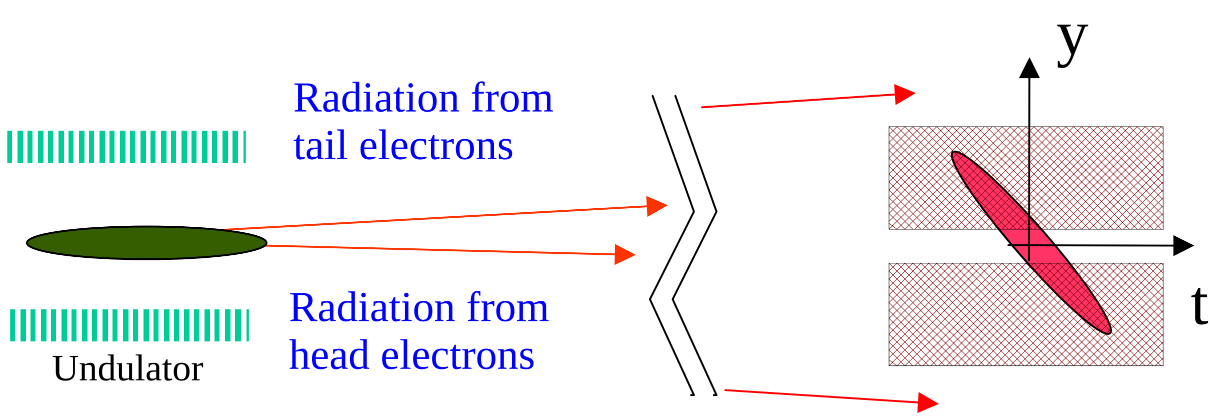
Transverse RF kick concept



Cavity frequency is harmonic *h* of ring rf frequency

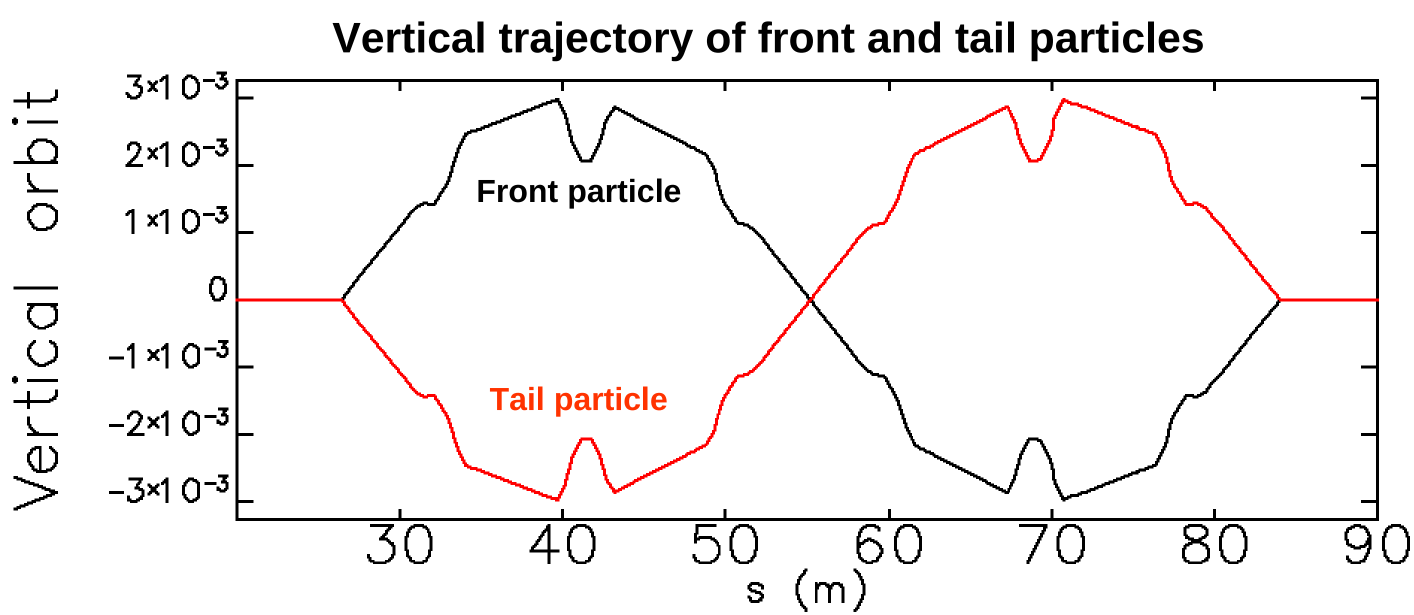
Ideally, second cavity exactly cancels effect of first if phase advance is n*180 degrees



Pulse can be sliced or compressed with asymmetric cut crystal

Beam trajectories

- After kick in the first cavity, front particles start moving up and tail particles start moving down with amplitude proportional to their longitudinal position in the bunch
- In an ideal linear system with zero energy spread, the second cavity completely cancels vertical motion of all particles



Sources of emittance increase

As was demonstrated in M. Borland's presentation this morning, the sources of emittance increase are

- **Sextupole fields**
- **Chromaticity and energy spread**
- **Momentum compaction and energy spread (minor)**

Sextupoles could be turned off between cavities, but that emphasizes the growth due to chromaticity

Sextupole field

Expression for magnetic field in sextupole:

$$B_y = \frac{1}{2} S \left(x^2 - y^2 \right),$$

$$B_x = S\, xy$$

For a particle traveling with non-zero vertical trajectory *y_o*:

$$B_y = \frac{1}{2} S \left(x^2 - y^2 \right) + S\, y_0\, y - \frac{1}{2} S\, y_0^2,$$

$$B_x = S\, xy - S\, y_0\, x$$

Additional fields:

- **First order – skew quadrupole**
- **Second order – dipole**

First order: Coupling

- **The degree of coupling depends on the tunes and the coupling coefficient:**

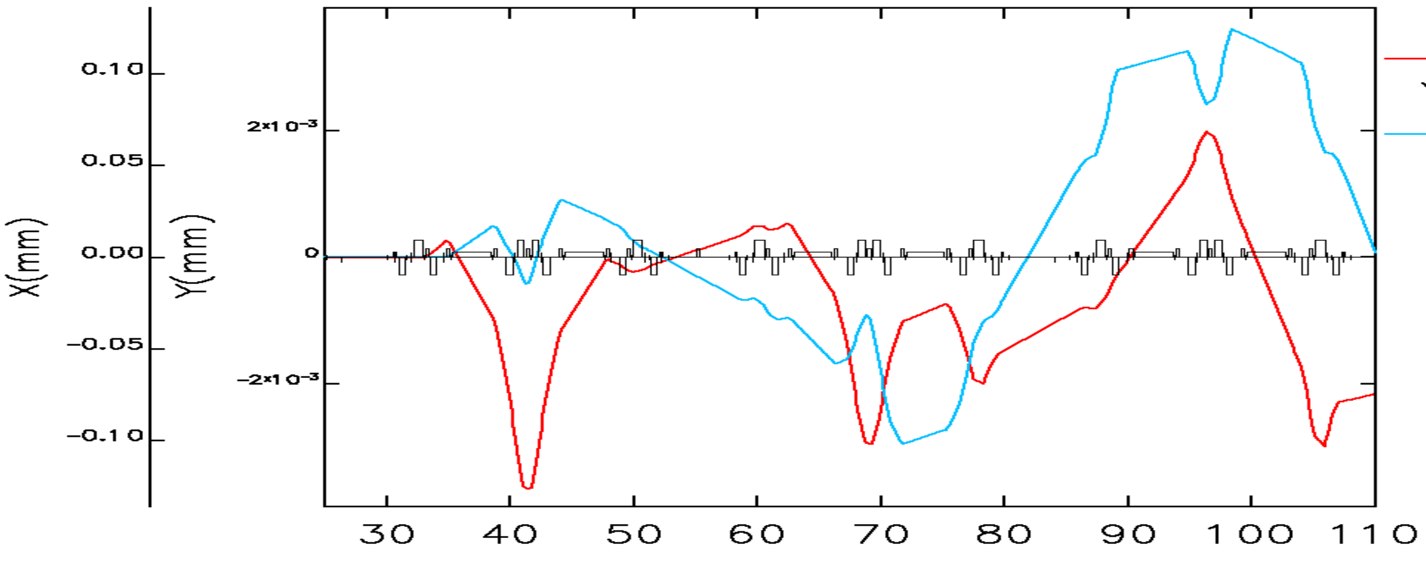
$$\kappa_q = \frac{1}{2\pi}\int_0^C \mathrm{K}_s \sqrt{\beta_x \beta_y}\, e^{i\Psi_q} ds \,,$$

$$\Psi_q = \psi_x - \psi_y - (\nu_x - \nu_y - q)\theta$$

- **If skew quadrupole is located in non-zero dispersion location, it also generates vertical dispersion**

Second order: dipole kick

- In the first sextupole, a particle experiences only vertical dipole kick, which generates horizontal trajectory
- At the second sextupole, the particle has both vertical and horizontal coordinates and experiences both horizontal and vertical dipole kick



Emittance increase with sextupoles

Emittance increase in the presence of sextupoles comes from:

- **Vertical emittance increase due to coupling (linear with rf kick amplitude)**
- **Vertical and horizontal emittance increase due to small dipole kicks on each turn (non-linear with rf kick amplitude)**

- **By optimizing sextupole strengths between cavities we can exactly cancel coupling term and minimize effect of dipole kicks within bunch**

Sextupole optimization for APS using `elegant` (2-sector case)

- **Variables:**
  - All sextupoles between cavities grouped into seven families in a symmetric fashion
- **Variable limits:**
  - Maximum sextupole gradient is increased by 25%
  - Sextupole signs are kept constant (to decrease symmetry breaking)
- **Constraints (calculated on vertical trajectory for a particle at $3\sigma_t$):**
  - Minimize 17^{th} coupling harmonic (ν_x=36.20, ν_y=19.26)
  - Minimize residual amplitude in X and Y
  - Minimize residual vertical dispersion
  - Compensate horizontal and vertical chromaticity

Alternative optimization procedure

- **Direct optimization based on one-pass multi-particle tracking through the deflecting section with deflecting cavities on using `elegant`**
- **Constraints are:**
  - Minimize 17th coupling harmonic
  - Minimize single-pass increase of vertical and horizontal emittances
  - Compensate chromaticity

- **Minimizing single-pass emittance increase is essentially minimizing dipole kicks for all amplitudes. The direct optimization gives 10% better result on multi-pass emittance blow-up minimization.**

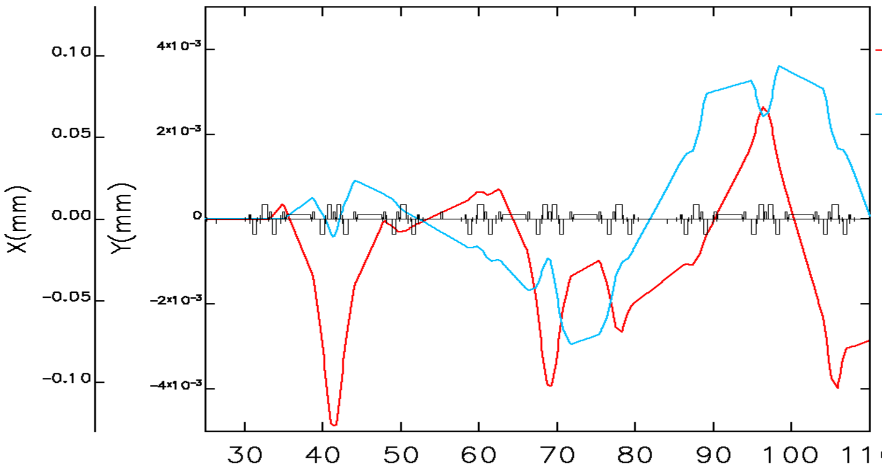
Optimization results for 6 MV rf voltage

- Table below shows improvement of constraints after optimization. Emittance blow-up compensation is not perfect within accepted limitations

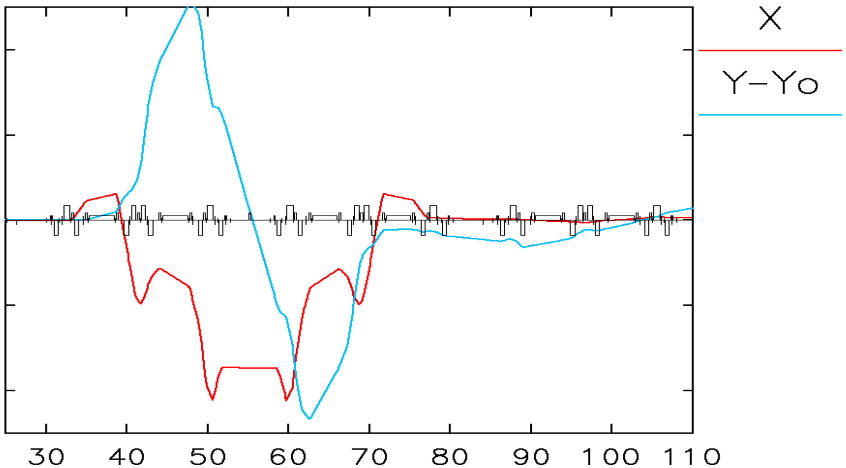
	Normal sextupoles	Optimized sextupoles
κ_{17}	$2.2 \cdot 10^{-2}$	$7.9 \cdot 10^{-3}$
X ($m^{1/2}$)	$1.8 \cdot 10^{-5}$	$4.5 \cdot 10^{-7}$
Y ($m^{1/2}$)	$8.9 \cdot 10^{-7}$	$8.4 \cdot 10^{-8}$
ε_x (nm rad)	10.1	3.0
ε_y (pm rad)	600	80

Optimization results (nonlinear orbit)

Nominal sextupoles

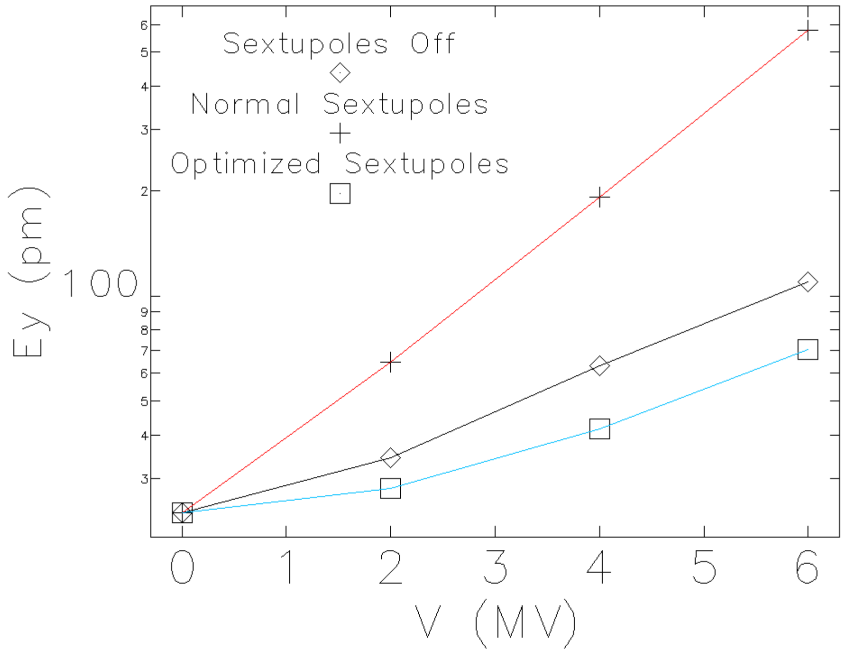


Optimized sextupoles



Optimization results (resulting vertical emittance)

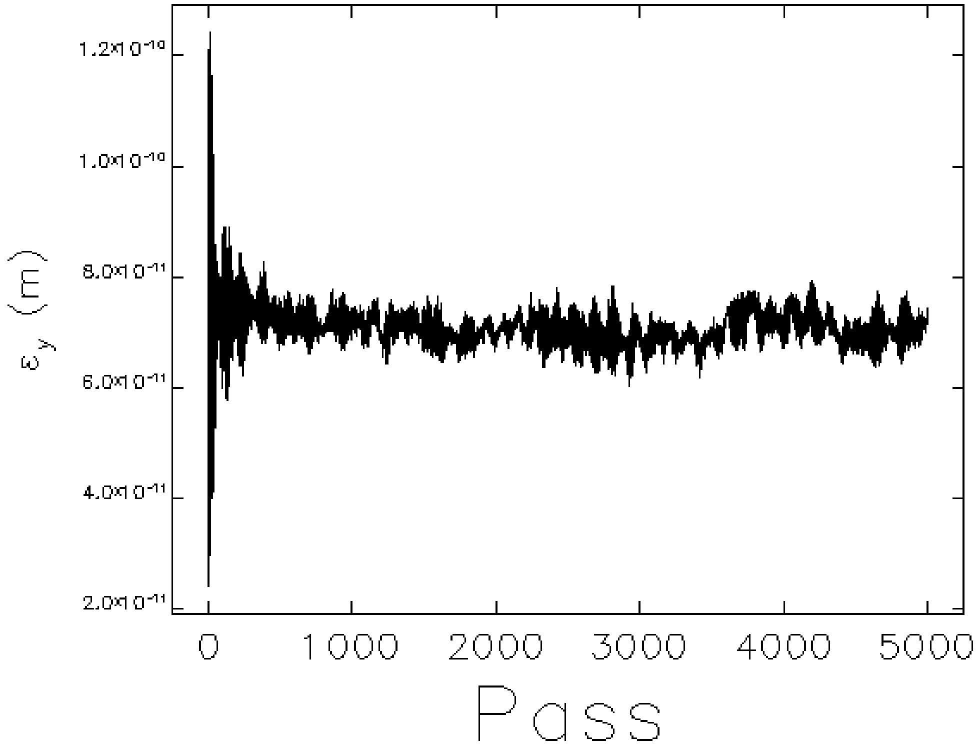
Comparison of the three sextupole schemes (no synchrotron radiation)



Example of lower initial coupling with optimized sextupoles

Optimization results (3)

Previous studies have shown that synchrotron radiation can greatly affect the tracking results. Here we show that simulation with synchrotron radiation does not change the results.



Additional options (1)

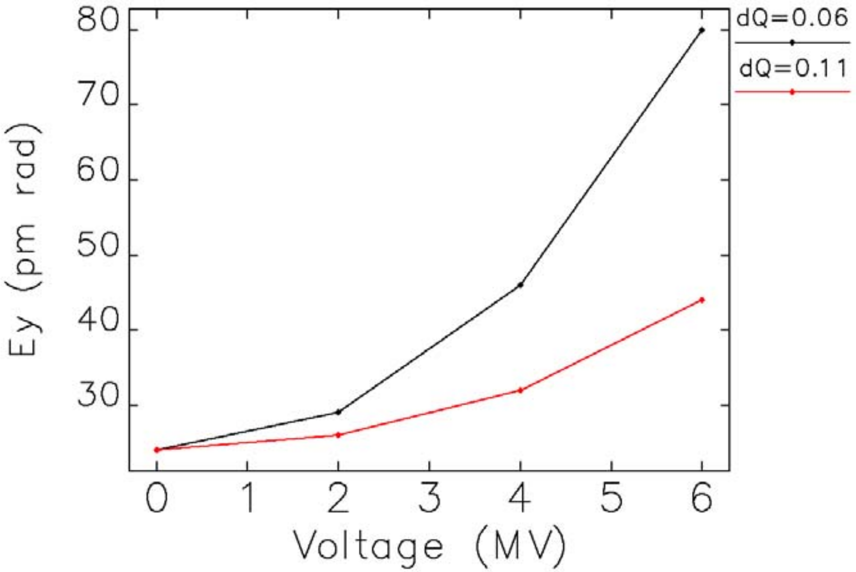
- Within realistic limits, sextupole optimization does not completely eliminate emittance blow-up
- There is an additional simple option to decrease coupling effect: distance between horizontal and vertical betatron tunes

Plot to the right shows emittance increase for nominal tunes

ν_x=36.20, ν_y=19.26

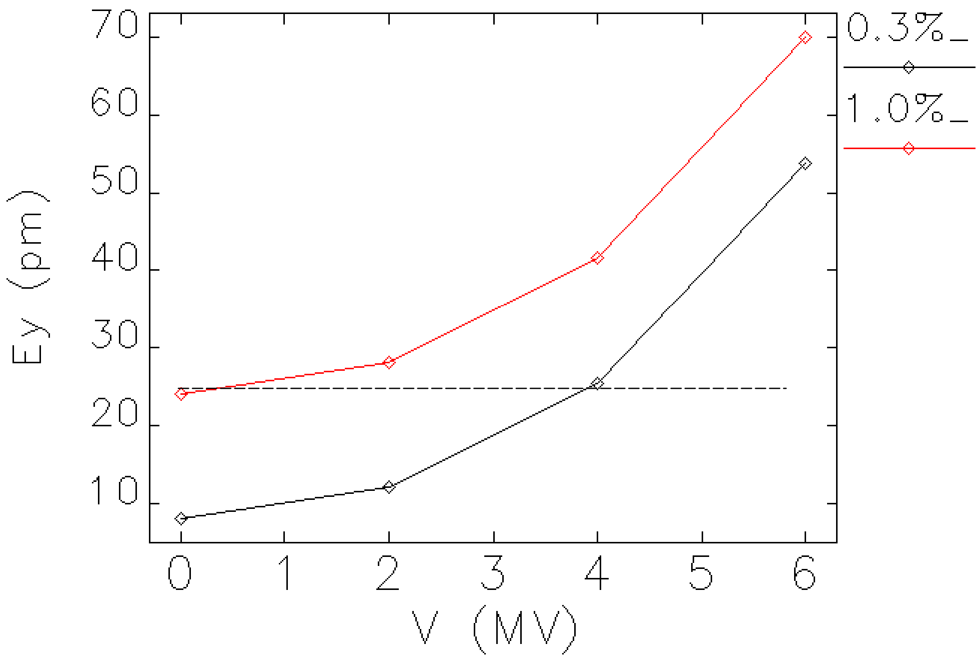
and new tunes

ν_x=36.16, ν_y=19.27

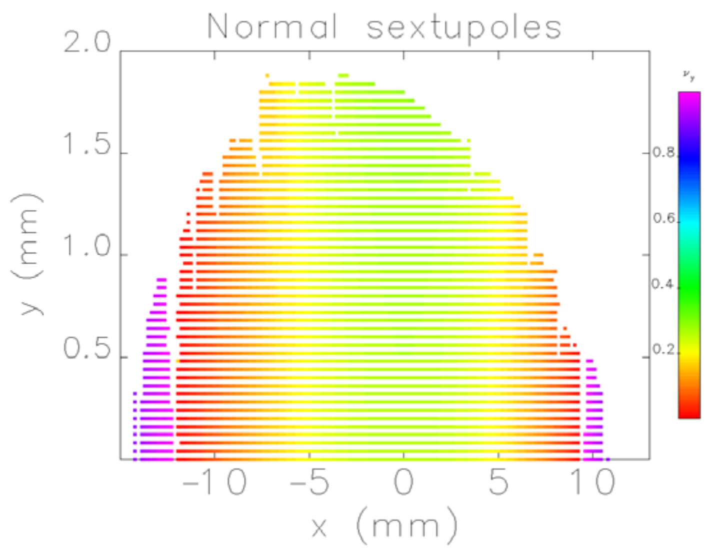


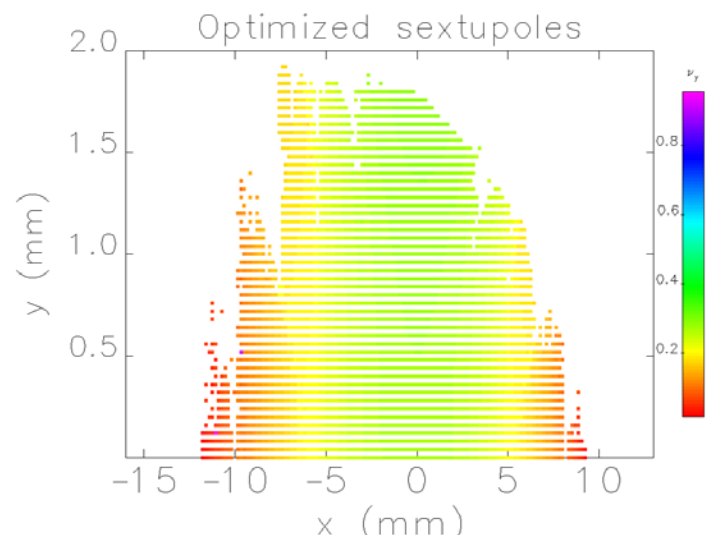
Additional options (2)

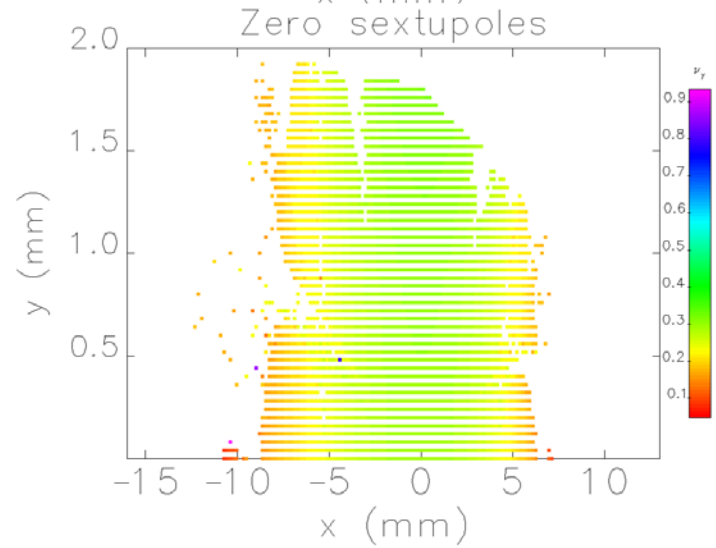
- Minimize coupling before the cavities are turned on. For APS, operating coupling is 1% or 25 pm rad emittance. Using skew quadrupoles around the ring, the coupling can be minimized to 0.3% or 8 pm rad emittance



Dynamic aperture comparison







Lattice without errors

500 turns tracking

Color indicates vertical tune

Expansion to more than 2 sectors

- Optimization of sextupoles opens possibility to increase the number of sectors that could benefit from the compression scheme

Number of sectors	Vertical emittance
2	70 pm
3	59 pm
4	41 pm

- Vertical emittance blowup is no longer a limitation. Instead, new limit would be dynamic aperture decrease

Conclusions

- **Due to proper optimization of sextupole strength, the vertical emittance increase is no longer a limiting issue for this scheme.**

- **It seems possible to increase the number of sectors between cavities to more than two. That would require additional dynamic aperture study, which is underway.**